Multimodal Variational Analysis of Uniaxial Waveguide Discontinuities

Jun-Wu Tao, *Member, IEEE*, and Henri Baudrand, *Senior Member, IEEE*

***Abstract*—A unified multimodal variational formulation is described for the characterization of uniaxial waveguide discontinuities. A variational form is obtained by using a self-adjoint susceptance operator which is defined in terms of two eigenmode sets corresponding to the two constituent waveguides. By making use of the similarity between the field and network theory, the final admittance matrix will be of small size owing to the use of the "accessible" and "localized" modes. This formulation leads to an appreciable reduction in the computation time and computer memory space and facilitates in this way the use of the optimization process on personal computers and workstations. Examples are given for both homogeneous and inhomogeneous waveguide discontinuity problems, showing good agreement with the experiments. An evanescent-mode ridge-waveguide filter has been designed on this theoretical basis with success.**

I. Introduction

STUDIES of discontinuity problems in waveguides are particularly important for the design of a number of microwave components. It is essential to be able to predict with accuracy the frequency-dependent behavior of these devices, especially concerning the discontinuities involved. This necessity derives from the increasing difficulty of post-manufacture adjustments in the millimeter-wave range, where one can hardly introduce tuning posts or screws in the metallic waveguide structure, and the modification of many microwave integrated circuits (MIC) structures is simply not possible.

A typical discontinuity problem often encountered in the design of phase shifters, impedance transformers, and filter structures is the uniaxial discontinuity. The variational nature of the formulation was the main concern of the first contributors to field and network theory and still attracts attention today. In the early work of Schwinger and his colleagues, which has been reported by Marcuvitz [1] and also in the standard textbook of Collin [2], stationary solutions were proposed for the equivalent circuit elements of isolated discontinuities, either by simple variational methods or by quasi-static methods. The results are fairly accurate below a certain frequency limit. Close-form expressions have even been derived for some discontinuity problems in the quasi-static case. Application of these methods has been extended to the discontinuity problems for which analytical expressions of the propagation characteristics do not exist, such as discontinuities in finned and ridge waveguides [3]–[5]. For the characterization of interacting discontinuities, many computer-oriented methods propose the modal expansion of trial fields on two finite sets of eigenmodes, each corresponding to a constituent waveguide; the inversion of the resulting matrix equation leads to the scattering matrix for the discontinuity considered [6]. This straightforward mode-matching technique has been applied to a large number of discontinuity problems for which the constituent waveguides are either homogeneous or inhomogeneous [7]–[17]. Proper convergence is not always ensured with the increasing order of double modal expansions as invoked by Mittra *et al.* [18], and the matrix treatment generally requires mainframe computer power. More analytical methods, such as the modified residue calculus technique [19], do not suffer from the relative convergence phenomenon and provide accurate results. Unfortunately, only limited applications, often with only x-direction discontinuities, are available in the literature [20]–[22]. Some studies have been carried out by an extended spectral-domain method, as in [23].

In his studies of interacting irises and steps in a homogeneous uniform waveguide [24]–[26], Rozzi has developed the variational method described by Collin [2] and has extended it to more than one interacting discontinuity. By making use of the similarity between eigenmodes and transmission lines and by examining the behavior of higher order eigenmodes in the neighborhood of discontinuities, he has introduced the notion of "accessible" and "localized" modes. A stationary expression for the reactance matrix corresponding to "accessible modes" has been obtained which requires manipulations with small-order matrices only. The same process has been applied to the step discontinuities in planar dielectric waveguides [27]–[28] and in microstrip [29].

In this paper, we propose a unified multimodal variational formulation for uniaxial discontinuity problems. This formulation, already applied to homogeneous discontinuity problems [30]–[33], has been extended to the inhomogeneous case. The discontinuities are assumed lossless and reciprocal, so a self-adjoint susceptance operator can be defined at the transverse discontinuity plane, which relates the arithmetic sum of the transverse magnetic field to the aperture (nonmetallic part of the cross section) electric field, which will be the only unknown in the resultant stationary expression. The aperture field is then expanded in terms of an appropriate eigenfunction set which should satisfy the boundary conditions. The corresponding admittance matrix of the discontinuity is then obtained by applying the nontrivial solution condition and by making use of the similarity between field theory and network theory.

If no distinction is made between the accessible and localized modes as described by Rozzi [24]–[26], [29], the admit-

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The authors are with the Laboratoire de Microondes, ENSEEIHT, 2, rue Charles Camichel, 31071 Toulouse, France.
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tance matrix will be of infinite order since all higher order modes will correspond to an "accessible port." By appropriately introducing this notion in the self-adjoint susceptance operator formulation, we obtain separately the accessible part and the localized part. The admittance matrix obtained in this way then corresponds to the accessible modes and is of small size. Only one matrix needs to be inverted, and the dimensions of this matrix depend on the aperture field expansion and may be of moderate size owing to the stationary nature of this formulation. Furthermore, this matrix presents the influence of the localized modes in infinite series form. The convergence of these series may be accelerated by an appropriate transform such as that of Schwinger.

As the resultant admittance matrix is often of small size, depending only on the coupling between adjacent discontinuities, this formulation is suitable for integration in a microcomputer- or workstation-based computer-aided-design package for predicting the behavior of multiple discontinuities, especially when the overall scattering parameters will be optimized in microwave device design procedures. Analysis has been carried out for finned and ridged waveguide discontinuities, as well as for dielectric-loaded waveguide discontinuities. Very good agreement has been observed between the predicted and measured results for both the homogeneous and inhomogeneous cases. An optimization design program has been developed on an IBM personnel computer by making use of this formulation. An evanescent-mode ridged waveguide low-pass filter has been designed in reasonable computation time and provides very good performance.

In the following section, we will first describe the unified multimodal variational formulation for the general case. Special consideration will then be given to completely isolated discontinuities and infinitely thin obstacles, as well as to two interacting discontinuities with z-direction symmetry. Application of this formulation will be demonstrated by practical studies.

II. Multimodal Variational Formulation

According to Harrington's "mode function," "mode voltage," and "mode current" concepts [34], the network expressions for transverse $\boldsymbol{E}$ and $\boldsymbol{H}$ fields in a reciprocal waveguide are given by

$$\boldsymbol{E}(x,y,z_0)=\sum_n v_n(z_0)\boldsymbol{E}_n(x,y) \tag{1a}$$

$$\boldsymbol{J}(x,y,z_0)=\sum_n i_n(z_0)\boldsymbol{J}_n(x,y). \tag{1b}$$

Here $v_n(z_0)$ and $i_n(z_0)$ correspond respectively to the sum and difference of incident and reflected wave amplitudes of the nth eigenmode at the reference plane z_0. The modal surface current density vector $\boldsymbol{J}_n$ is related to $\boldsymbol{H}_n$ by

$$\boldsymbol{J}_n=\boldsymbol{H}_n\times \boldsymbol{z} \tag{2}$$

$\boldsymbol{z}$ being the direction of propagation. Boldface italic letters are used for space vectors. The orthogonality relation for z-independent mode functions is given by

$$\langle \boldsymbol{J}_n,\boldsymbol{E}_m\rangle=\int(\boldsymbol{J}_n^*\cdot\boldsymbol{E}_m)\,dS=N_n\delta_{nm} \tag{3}$$

δ_{nm} being the Kronecker delta and N_n taking the values of 1, j, or $-j$, depending on whether the nth eigenmode is propagating or evanescent with capacitive or inductive behavior. $\boldsymbol{J}_n^*$ denotes the complex conjugate of $\boldsymbol{J}_n$. The inner dot product in (3) replaces the classical vector product due to the definition of $\boldsymbol{J}$.

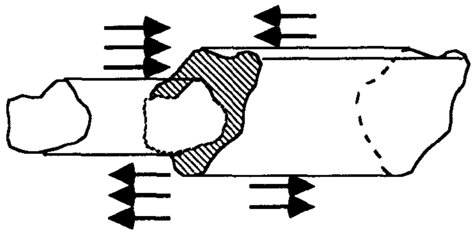


Fig. 1. Uniaxial waveguide discontinuity.

A normalized admittance coefficient $y_n(z_0)$ can be introduced at the reference plane so that

$$i_n(z_0)=y_n(z_0)v_n(z_0). \tag{4}$$

By defining the eigenmode admittance operator $\hat{Y}_n$ as below,

$$\hat{Y}_n\boldsymbol{F}=\boldsymbol{J}_n\langle \boldsymbol{J}_n,\boldsymbol{F}\rangle/N_n=(\boldsymbol{J}_n/N_n)\int(\boldsymbol{J}_n^*\cdot\boldsymbol{F})\,dS \tag{5}$$

we can extract the expansion coefficient v_n by applying $\hat{Y}_n$ to (1a) and introducing it into (1b) via relation (4). The transverse $\boldsymbol{E}$ and $\boldsymbol{H}$ fields are then related by the following expression:

$$\boldsymbol{J}=\left(\sum_n y_n\hat{Y}_n\right)\boldsymbol{E}=\hat{Y}\boldsymbol{E}. \tag{6}$$

$\hat{Y}$ is defined as the admittance operator associated with a reciprocal waveguide.

In a uniaxial waveguide discontinuity problem of the type shown in Fig. 1, owing to the possible interaction with the neighboring discontinuities, both first and higher order eigenmodes can be incident at each side of the discontinuity plane, and the relation (6) holds for both waveguides. The arithmetic sum of surface current densities will be zero in the aperture (the nonmetallic part of the cross section) according to the continuity relation of the transverse $\boldsymbol{H}$ field, since the incidence is assumed to be along the negative z direction in the right-hand waveguide. By making use of the transverse $\boldsymbol{E}$ field continuity, the admittance operator $\hat{Y}$ associated with the discontinuity problem will simply be the sum of two admittance operators associated with each waveguide, and the following relation holds in the aperture region:

$$\hat{Y}\boldsymbol{E}=0 \tag{7}$$

with $\hat{Y}$ given by the following relationship:

$$\hat{Y}=\sum_i y_i^{(1)}\cdot\hat{Y}_i^{(1)}+\sum_j y_j^{(2)}\cdot\hat{Y}_j^{(2)}. \tag{8}$$

When the accessible ports are terminated by purely reactive admittances, we have $\hat{Y}=j\hat{B}$, $\hat{B}$ being the self-adjoint susceptance operator. The associated stationary expression will be given by

$$f=\langle \boldsymbol{E},\hat{B}\boldsymbol{E}\rangle. \tag{9}$$

The aperture field $\boldsymbol{E}$ can be expanded over an appropriate eigenfunction basis $G=\{\boldsymbol{g}_n\}$ which satisfies the boundary conditions. The susceptance operator $\hat{B}$ is then expressed by a corresponding matrix $\bar{\bar{B}}$ with a general term in the following form:

$$(\bar{\bar{B}})_{mn}=\langle \boldsymbol{g}_n,\hat{B}\boldsymbol{g}_m\rangle. \tag{10}$$

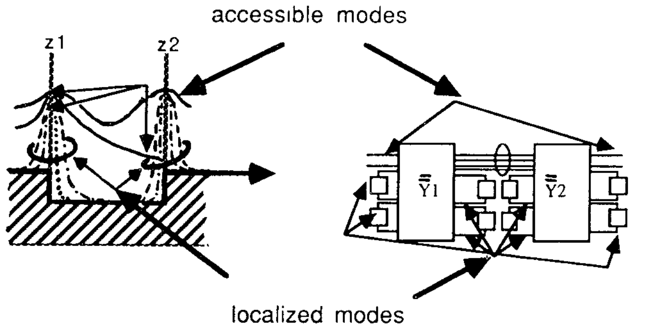


Fig. 2. Behavior of accessible and localized modes in the vicinity of discontinuities and equivalent network representation.

The minimization of (9) leads to the following homogeneous equation:

$$\sum_{n} B_{mn} \nu_n = 0 \tag{11}$$

ν_n being the nth expansion coefficient of the trial field E in the basis $\{g_n\}$.

When a higher order eigenmode, well below cutoff, is considered localized, no significant incidence will be observed, as shown in Fig. 2. The corresponding mode excited by the adjacent discontinuity is completely attenuated before reaching the discontinuity plane considered; the reduced admittance coefficient can then be taken as (-1) according to (4). By separating all localized modes from the accessible ones, the susceptance operator is reformulated and the corresponding matrix elements can be rewritten as

$$jB_{mn} = \sum_{i-1}^{L} y_i^{(1)} \langle g_m, \hat{Y}_i^{(1)} g_n \rangle + \sum_{j=1}^{K} y_j^{(2)} \langle g_m, \hat{Y}_j^{(2)} g_n \rangle - \sum_{i=L+1}^{\infty} \langle g_m, \hat{Y}_i^{(1)} g_n \rangle - \sum_{j=K+1}^{\infty} \langle g_m, \hat{Y}_j^{(2)} g_n \rangle. \tag{12}$$

By defining the following matrices:

$$(\bar{\bar{Q}})_{mn} = (-j) \left\langle g_m, \left(\sum_{i=L+1}^{\infty} \hat{Y}_i^{(1)} + \sum_{j=K+1}^{\infty} \hat{Y}_j^{(2)} \right) g_n \right\rangle \tag{13}$$

$$(\bar{\bar{U}}^{(\nu)})_{ml} = \langle g_m, J_l^{(\nu)} \rangle, \qquad l = 1, 2, \cdots L \text{ or } K \tag{14}$$

(11) can be reorganized as

$$\left(\sum_{\nu=1}^{2} \bar{\bar{U}}^{(\nu)} j \bar{\bar{B}}_{\text{diag}}^{(\nu)} \bar{\bar{N}}^{(\nu)^{-1}} \bar{\bar{U}}^{(\nu)t} \right) \bar{V} = -\bar{\bar{Q}} \cdot \bar{V}, \qquad \bar{V} = \begin{bmatrix} \nu_1 \\ \nu_2 \\ \vdots \end{bmatrix}. \tag{15}$$

$\bar{\bar{B}}_{\text{diag}}^{(\nu)}$ is the diagonal susceptance matrix with the nth diagonal element corresponding to $y_n^{(\nu)}$. $\bar{\bar{N}}^{(\nu)}$ is also diagonal with diagonal elements corresponding to the normalization constants given (3). A vector $\bar{T}$ depending only on the accessible modes can be defined by means of the following relationship:

$$\bar{T} = \begin{bmatrix} \bar{\bar{U}}^{(1)t} \\ \bar{\bar{U}}^{(2)t} \end{bmatrix} \cdot \bar{V} \tag{16}$$

and the following homogeneous equation of order $L + K$ holds:

$$\left(j \bar{\bar{\Gamma}} \cdot \bar{\bar{N}}^{-1} \bar{\bar{B}}_{\text{diag}} + \bar{\bar{I}} \right) \cdot \bar{T} = \bar{0} \tag{17}$$

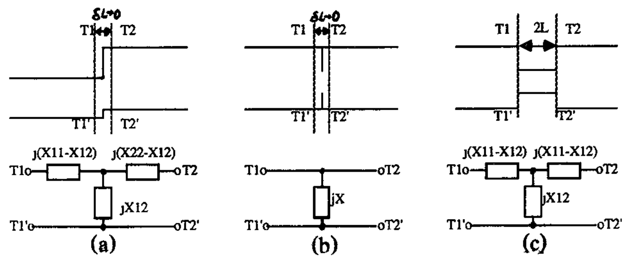


Fig. 3. Particular cases and their equivalent circuits: (a) isolated junction; (b) thin obstacle; (c) thick obstacle.

where

$$\bar{\bar{\Gamma}} = \begin{bmatrix} \bar{\bar{U}}^{(1)t} \bar{\bar{Q}}^{-1} \bar{\bar{U}}^{(1)} & \bar{\bar{U}}^{(1)t} \bar{\bar{Q}}^{-1} \bar{\bar{U}}^{(2)} \\ \bar{\bar{U}}^{(2)t} \bar{\bar{Q}}^{-1} \bar{\bar{U}}^{(1)} & \bar{\bar{U}}^{(2)t} \bar{\bar{Q}}^{-1} \bar{\bar{U}}^{(2)} \end{bmatrix} \tag{18}$$

and $\bar{\bar{I}}$ is the unit matrix. The nontrivial solutions of (17) can be obtained only by equating $|j\bar{\bar{\Gamma}} \cdot \bar{\bar{N}}^{-1} \bar{\bar{B}}_{\text{diag}} + \bar{\bar{I}}|$ to zero. On the other hand, by making use of (4), we can define the discontinuity admittance matrix "visible" to the adjacent discontinuities so that

$$\bar{I} = \bar{\bar{B}}_{\text{diag}} \bar{V} = \bar{\bar{Y}} \cdot \bar{V}. \tag{19}$$

The nontrivial solution of (19) requires that the determinant $|\bar{\bar{Y}} - \bar{\bar{B}}_{\text{diag}}| = 0$ and one can then identify the admittance matrix $\bar{\bar{Y}}$ as well as the impedance matrix $\bar{\bar{Z}}$ corresponding to the accessible ports:

$$\bar{\bar{Y}} = j \bar{\bar{N}} \cdot \bar{\bar{\Gamma}}^{-1} \qquad \bar{\bar{Z}} = -j \bar{\bar{\Gamma}} \cdot \bar{\bar{N}}^{-1}. \tag{20}$$

The corresponding scattering matrix, if needed, can be derived according to many standard textbooks, for instance that of Harrington [34].

When a multiple discontinuity problem is considered, we can transform the global boundary problem to the network problem corresponding to a series of cascaded networks. The accuracy of individual discontinuity solution depends only on the trial aperture field expansion, regardless of the number of accessible modes, whereas the accuracy of the overall characteristics depends strongly on this number. For a uniform waveguide of given length L which relates two adjacent discontinuities, the number of modes accessible to both discontinuities will be determined from the attenuation coefficients $e^{-\alpha_n L}$.

In some special cases where the discontinuities can be considered fairly "isolated," the accessible mode number will be determined according to their equivalent circuits, as discussed below.

III. Particular Cases

A. Isolated Discontinuities (Fig. 3(a))

The equivalent network problem will be one-port or two-port, depending on whether both waveguides are propagating or not. Considering first the two-port case, we have

$$\bar{\bar{Z}} = j \begin{bmatrix} x_{11} & x_{12} \\ x_{12} & x_{22} \end{bmatrix} \tag{21}$$

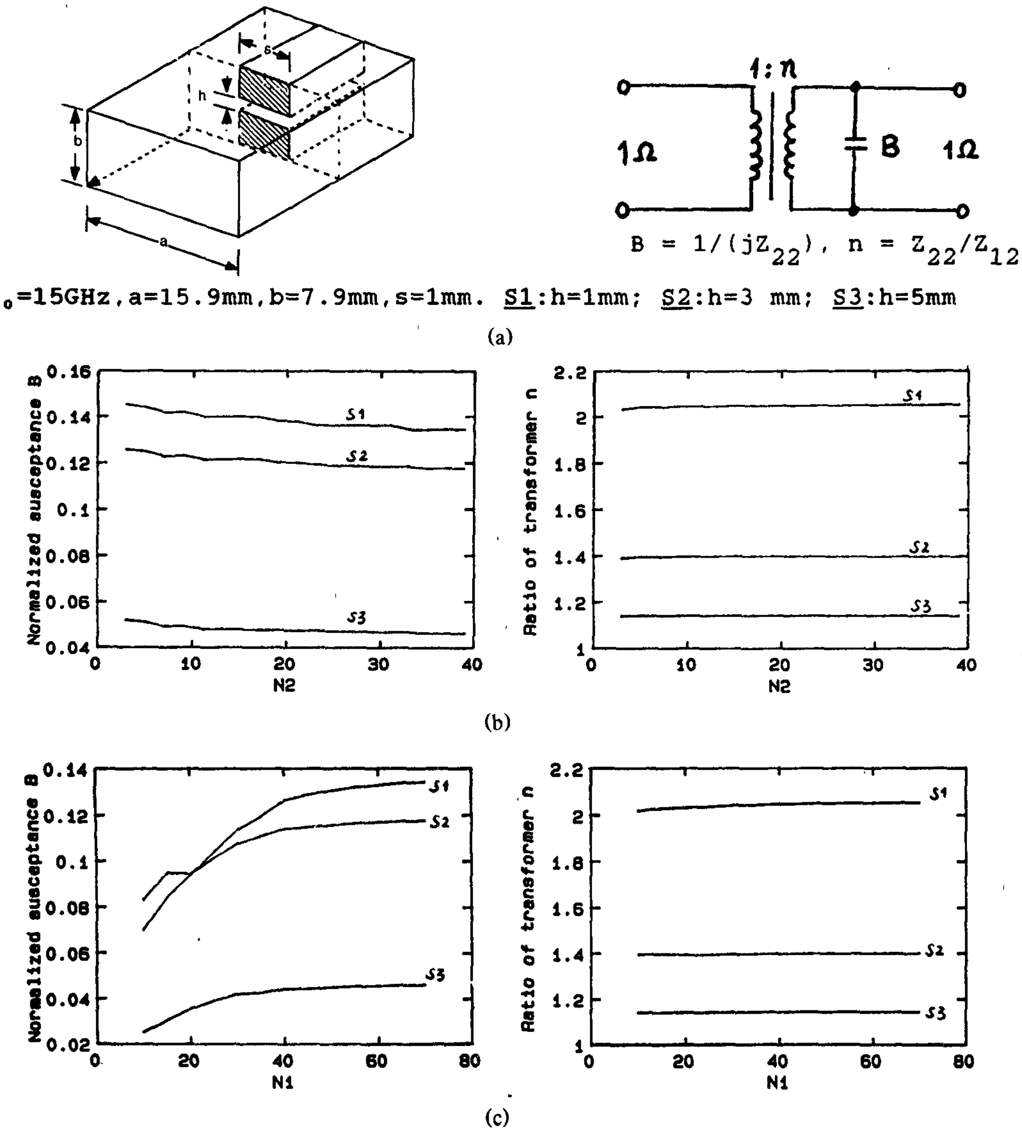


Fig. 4. (a) Double ridged waveguide–empty waveguide discontinuity and its equivalent circuit [1]. (b) Equivalent circuit elements versus aperture field expansion number $N2$ ($N1 = 70$). (c) Equivalent circuit elements versus empty waveguide eigenmodes number $N1$ ($N2 = 35$).

with

$$x_{11} = -\langle J_1^{(1)}, \bar{G}^t\rangle \cdot \bar{\bar{Q}}^{-1}\langle \bar{G}, J_1^{(1)}\rangle$$
$$x_{22} = -\langle J_1^{(2)}, \bar{G}^t\rangle \cdot \bar{\bar{Q}}^{-1}\langle \bar{G}, J_1^{(2)}\rangle$$
$$x_{12} = -\langle J_1^{(1)}, \bar{G}^t\rangle \cdot \bar{\bar{Q}}^{-1}\langle \bar{G}, J_1^{(2)}\rangle$$
$$\bar{G}^t = (g_1 g_2 g_3 \cdots g_p \cdots).$$

We can see that $\bar{\bar{Z}}$ is symmetric owing to the symmetry of $\bar{\bar{Q}}$. This verifies the reciprocity property assumed at the beginning. Moreover, if all higher order modes are below cutoff, $\bar{\bar{Z}}$ will be purely imaginary.

$\bar{\bar{U}}^{(2)}$ will be an empty matrix for the one-port case so that the discontinuity will be characterized only by its equivalent impedance jx_{11}, assuming, with no loss of generality, that the right-hand waveguide is below cutoff. The classical stationary expression of [5] may be found by setting $\{E_n^{(2)}\}$ as eigenfunctions and by considering only the first incident mode since $\bar{\bar{Q}}$ is then a diagonal matrix.

B. Infinitely Thin Obstacle (Fig. 3(b))

Any infinitely thin obstacle will be characterized only by a shunt impedance (jx) as shown Fig. 3(b). In fact, as the left-hand and right-hand waveguides are identical, we obtain from (21) that $x_{11} = x_{22} = x_{12} = x$. The network equation is then satisfied.

C. Thick Obstacle (Fig. 3(c))

When the discontinuity problem has geometrical symmetry, the solution can be split into even and odd parts by introducing respectively a magnetic and an electric wall at the symmetry plane, as described for the thick iris resolution by Collin [2] and Rozzi [24]–[26] respectively. The internal ports are, by definition, not accessible, but the eigenmodes cannot be considered localized as described above owing to the presence of the magnetic or electric walls at a nonnegligible distance. When these walls are introduced, the even and odd solutions for the reduced driving point impedances jx_{even} and jx_{odd} will be obtained in a manner similar to that

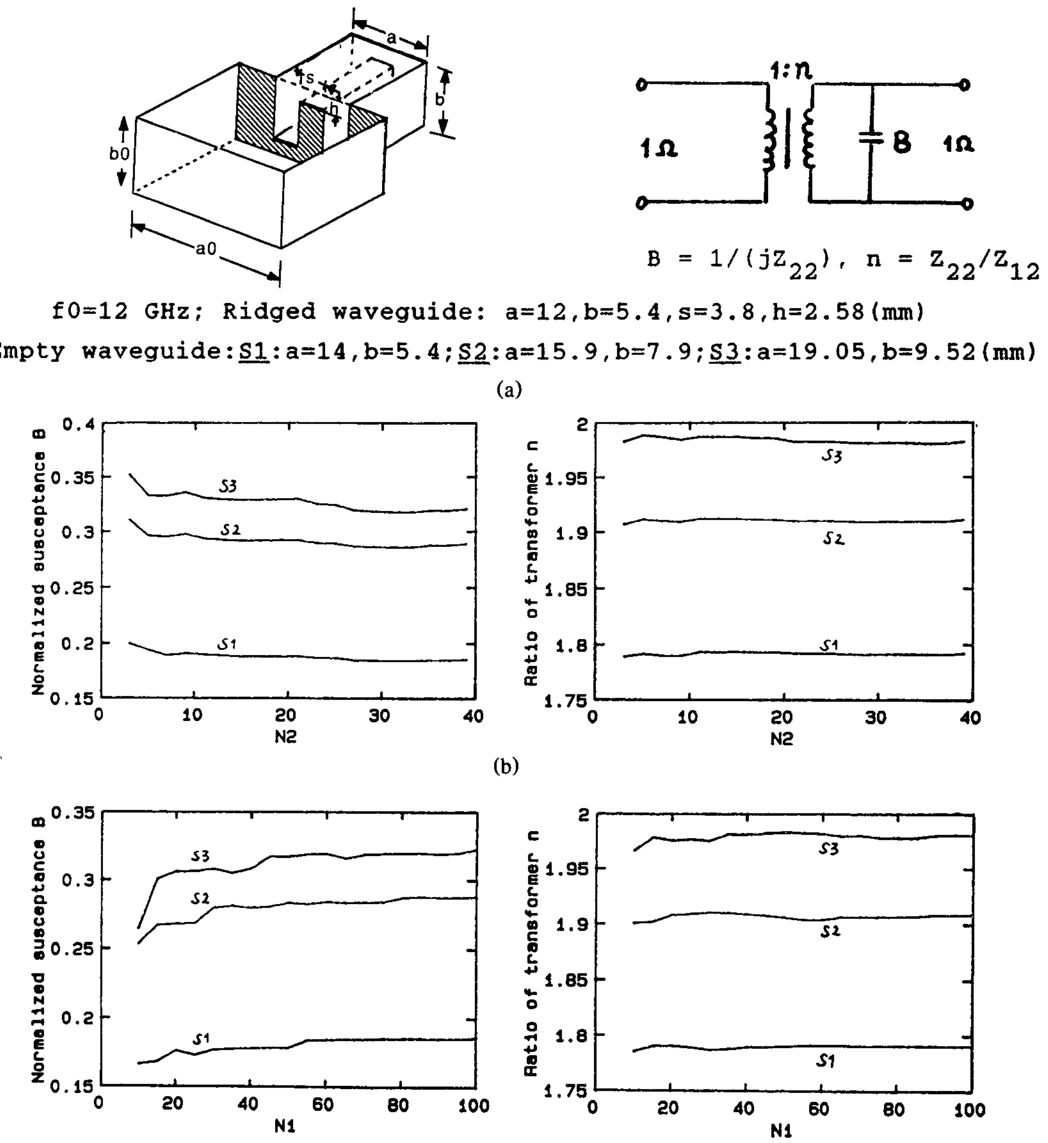


Fig. 5. (a) Ridged waveguide–empty waveguide discontinuity with different housing and its equivalent circuit [1]. (b) Equivalent circuit elements versus aperture field expansion number $N2$ ($N1 = 90$). (c) Equivalent circuit elements versus empty waveguide eigenmodes number $N1$ ($N2 = 35$).

TABLE I
COMPARISON OF MATRIX OPERATIONS REQUIRED FOR EACH TECHNIQUE

	Mode-Matching [14]		Multimodal Variational	
Matrix Inversions	(N,N)	C	(M,M) R,	$(n+m,n+m)$ C
Matrix Multiplications	$(N,N)\times(N,N)$	C	$(M,N-n)\times(N-n,M)$	R
	$(N,N)\times(N,M),(M,N)\times(N,N)$	C	$(n,M)\times(M,M)$	R
	$(M,N)\times(N,M)$	C	$(n,M)\times(M,n)$	R
	$(M,N)\times(N,M)$	C	$(n+m)\times(n+m)$	C

C—complex matrix; R—real matrix; n,m—numbers of accessible modes.

in the one-port isolated case simply by replacing the localized admittance coefficient (-1) with respectively $-j\cdot\cot(\beta_n L)$ and $j\cdot\tan(\beta_n L)$. According to [2], the resultant two-port network is then characterized by

$$jx_{11} = jx_{22} = j(x_{\text{even}} + x_{\text{odd}})/2$$
$$jx_{12} = j(x_{\text{even}} - x_{\text{odd}})/2. \quad (22)$$

IV. PRACTICAL APPLICATIONS

In this section, the multimodal variational formulation will be applied to both homogeneous and inhomogeneous discontinuity problems. The ridged rectangular waveguide discontinuities will be characterized by their equivalent circuit and scattering parameters, as will the dielectric-filled waveguide discontinuities.

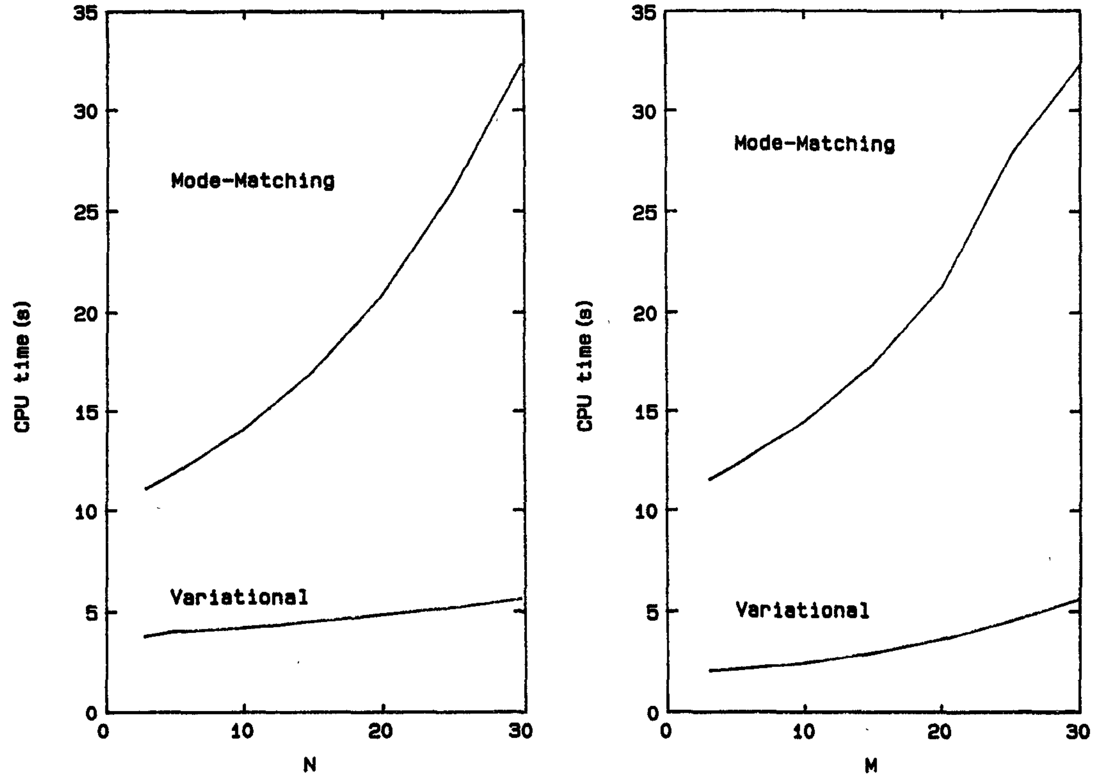


Fig. 6. Computation time versus empty waveguide eigenmode number N and that of aperture field expansion terms M with mode-matching techniques and multimodal variational formulation.

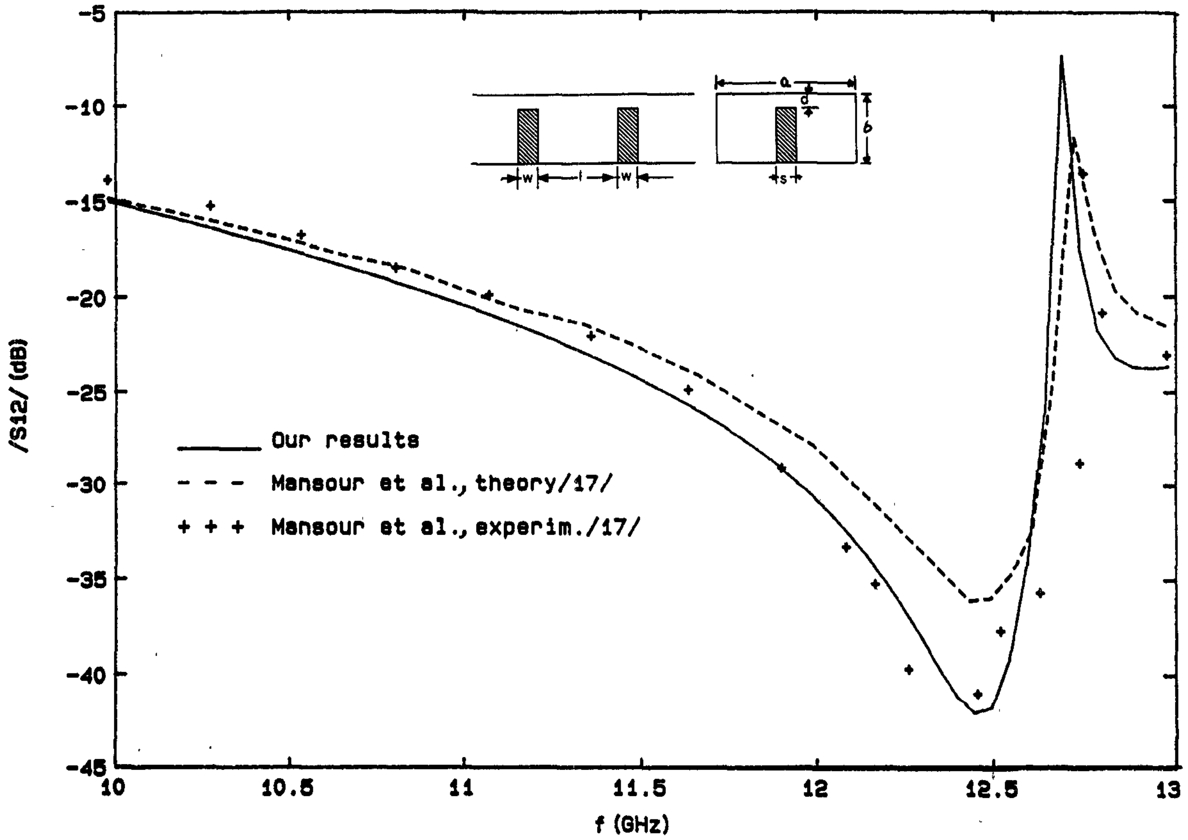


Fig. 7. Transmission coefficient of a cascaded E-plane ridged guide discontinuity: $a = 22.86$, $b = 10.16$, $d = 4.114$, $w = 1.524$, $l = 12.1192$ (mm).

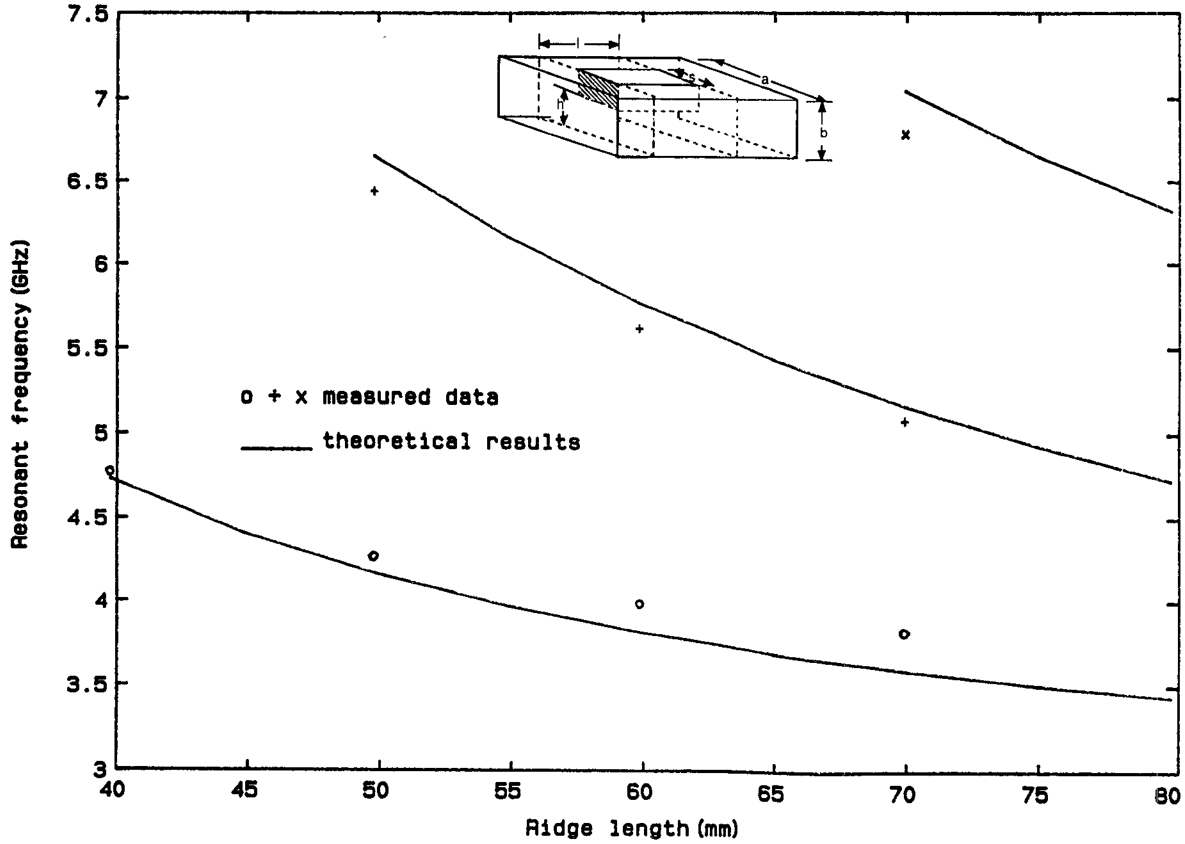


Fig. 8. Resonant frequencies versus ridge lengths. Comparison between the predicted and measured results: $a = 21$, $b = 9$, $s = 4.5$, $h = 1$ (mm).

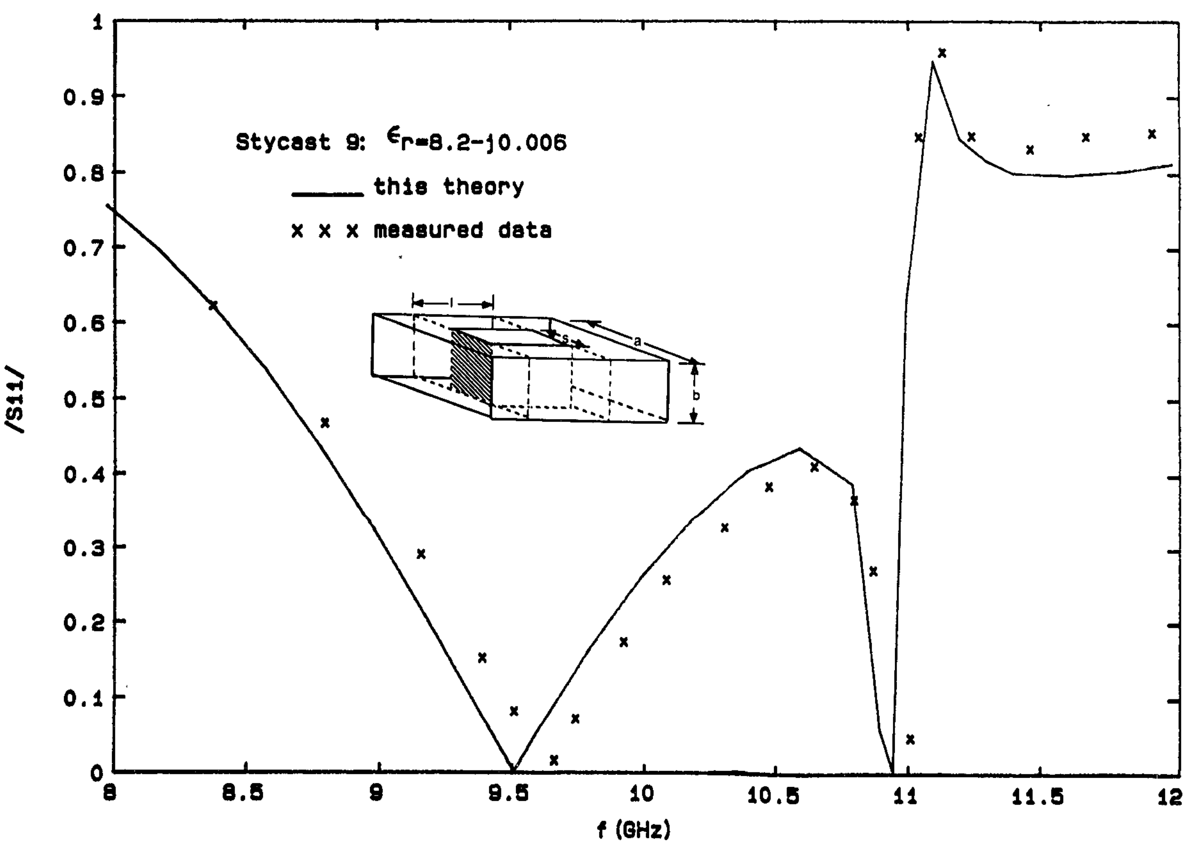


Fig. 9. Reflection coefficient of a dielectric obstacle in empty waveguide. Comparison with [39]: $a = 22.86$, $b = 10.16$, $s = 12$, $l = 6$ (mm); $\epsilon_r = 8.2 - j0.006$.

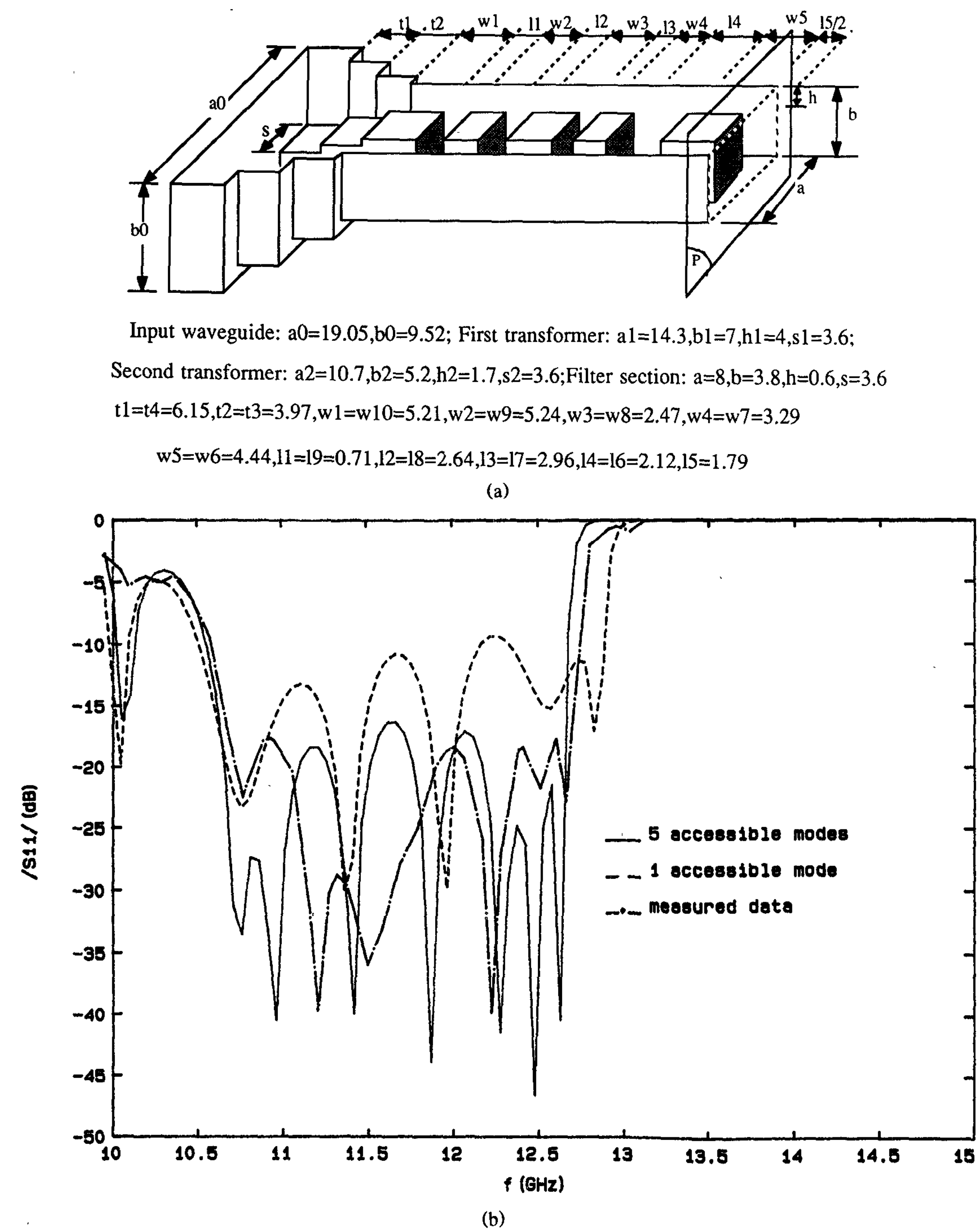
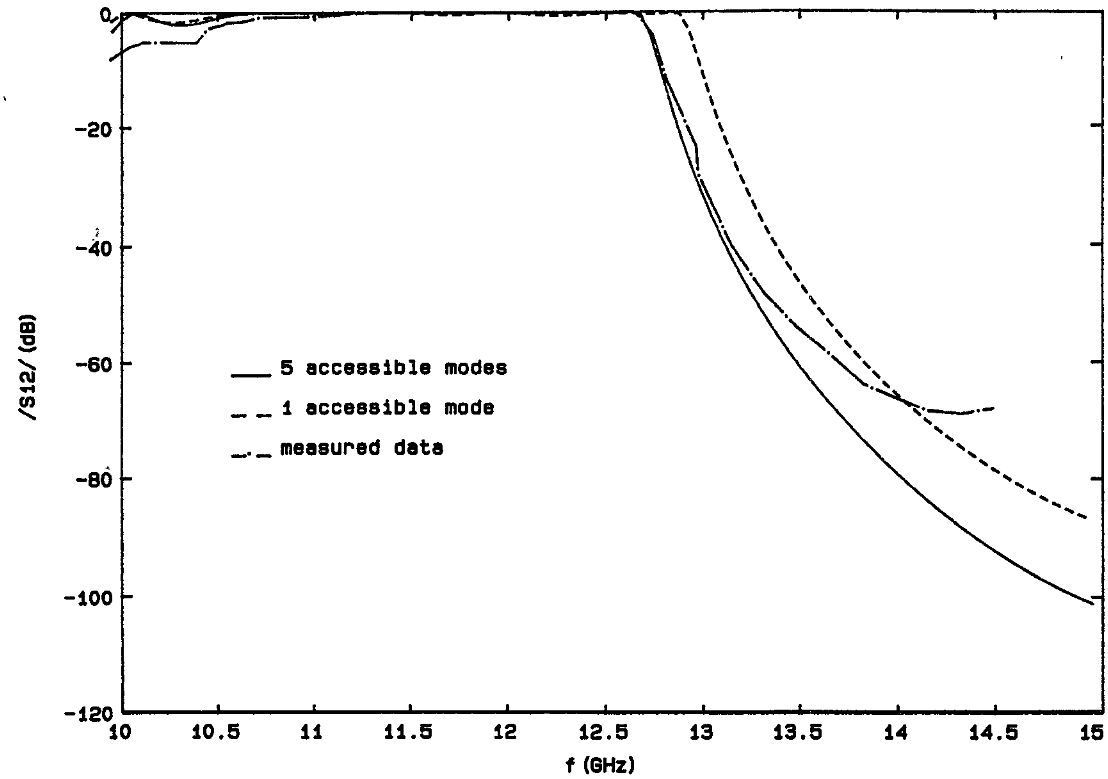


Fig. 10. (a) Evanescent-mode ridged waveguide low-pass filter. (b) Measured and calculated reflection coefficient.
(*Continued on next page*)

The propagation constants and the corresponding normalized field distribution are determined by applying rigorous analysis methods, the generalized transverse resonance method for the ridged waveguide analysis [35]–[37], and a transfer matrix formulation for the dielectric-loaded waveguide [38]. An accurate definition of field components is very important during the individual discontinuity characterization, which depends only on the aperture field expansion, as noted above. Although (9) is stationary in this field, a good choice of basis functions, one which approaches as closely as possible the real field distribution, will ensure faster convergence. When no better basis is available, the eigenmodes corresponding to the waveguide whose cross section coincides with the aperture are used. For this reason, the aperture field expansion of a ridged waveguide–empty waveguide junction is taken over the ridged waveguide eigenmodes basis.

First of all, the convergence test is carried out through the examples of discontinuities between the rectangular waveguide and the ridged waveguide. When no step of waveguide housing exists, as shown in Fig. 4(a), the equivalent circuit elements have been evaluated by varying the number of



(c)

Fig. 10 (*Continued*). (c) Measured and calculated transmission coefficient.

aperture expansion fields (Fig. 4(b)) and the number of empty waveguide eigenmodes (Fig. 4(c)). Different ridge heights have been used here and one can see the influence of the singularity at the metallic wedge. When the spacing between ridges is relatively large, the wedge phenomenon is less evident and only ten expansion terms of the aperture field and 40 eigenmodes in the empty waveguide are sufficient; in contrast, when this phenomenon becomes evident with small spacing, respectively 30 and 60 terms are needed. In both the small and the large spacing cases, the shunt capacitance and the transformer ratio vary uniformly with the increase of expansion terms. The discontinuity can then be considered "well behaved."

The housing step case is also treated with different step ratios (Fig. 5). We can see the well-known "relative convergence phenomenon" from parts (b) and (c) of the figure when the difference is great between the housing of the empty waveguide and that of ridged waveguide, by varying either the number of aperture field expansion terms or the number of empty waveguide eigenmodes. Generally speaking, more eigenmodes are needed for convergence in the double step housing case than in the nonstep case. This is why Bornemann *et al.* [10] did not judge it necessary for their study to increase the localized mode number since their discontinuity problem, taken as an example here, is a well-behaved one according to Fig. 4.

When the same numbers of accessible and localized modes are used, the multimodal variational formulation and the classical mode-matching method provide identical results, as noted in a previous paper [30]. However, by treating these two groups of modes differently as described above, the computation time is considerably reduced. For a lossless boundary-reduction discontinuity, the details of matrix operations required for both the mode-matching technique [14] and our formulation are shown in Table I, assuming that the aperture field expansion is taken over the basis of the eigenmodes of the right-hand waveguide. The ratio N/M, N and M being, respectively, the numbers of eigenmodes taken in the left- and right-hand waveguides, will be greater than 1 if the edge condition is considered [19].

Fig. 6 compares the CPU time relative to the scattering matrix computation needed for each technique with increasing total mode number on an IBM Personal Computer with an Intel-8087 arithmetic coprocessor. Three accessible modes are assumed for each waveguide, which will satisfy most of the interacting discontinuity problems. The difference, not very significant for limited mode number, increases rapidly in terms of the total number of modes. The same is true of the computer memory space, which is another important factor in programming on microcomputers and workstations.

The multimodal variational formulation has been applied to a cascaded E-plane ridged waveguide discontinuity problem. We have compared our results with those calculated and measured by Mansour *et al.* [17] in Fig. 7. Here 50 eigenmodes have been used in the rectangular waveguide and 16 in ridged waveguide. By using five accessible modes in the ridged waveguide and two in the rectangular one, we obtain better agreement with the measurement.

A ridged waveguide resonator is mounted by introducing a thick ridge in a below-cutoff waveguide. For a given ridge length, the resonance occurs when the total effective length, including the frequency-dependent end effect, is equal to a multiple of a half-wavelength. Fig. 8 shows that the three first predicted resonant frequencies agree well with the measurements carried out on an HP8510 vector network analyzer. No resonance will be observed above 7.14 GHz, corresponding to the cutoff frequency of the first mode in the empty waveguide.

The treatment of inhomogeneous discontinuity problems is illustrated by analyzing a finite-length dielectric obstacle in a rectangular waveguide. The calculated input reflection coefficient is compared with the measured value [39] in Fig. 9. The agreement is very good with ten TE_x modes of the dielectric-loaded waveguide.

A PC-based microwave filter design package has been developed with the multimodal variational formulation as the main analysis tool, owing to its numerical advantages. An optimization design has been taken for an eight-resonator evanescent-mode ridged-waveguide low-pass filter with two step transformers at the input and output terminals (Fig. 10(a)). About 6 h are required for the optimization procedure on an 8 MHz personal computer when using a direct search method [40]. The required filter performances are of maximum 1.3 SWR in the 10.7–12.7 GHz frequency range and more than 60 dB rejection at 14 GHz, which has been confirmed by the measurements (Fig. 10(b) and (c)). The predicted filter performances are also given by taking into account the influence of the number of accessible modes. The convergence of the overall scattering parameters is obtained with only three accessible modes, and five accessible modes are considered in the final analysis. The difference between the theoretical and measured results is due to the imperfections in the realization, which are not yet taken into account in our design package.

V. Conclusion

A unified multimodal variational formulation has been described for uniaxial waveguide discontinuities characterization. Both homogeneous and inhomogeneous waveguide discontinuity problems have been studied to illustrate the numerical advantages of this approach, which derive from the well-known "accessible" and "localized" modes. The handling of matrices of reduced size allows the development on the personal computer of such optimized computer-aided-design packages as that used for evanescent-mode ridged waveguide filters.

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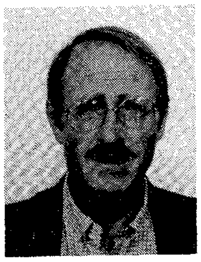
Jun-Wu Tao (M'91) was born in Hubei, China, in 1962. He received the B.Sc. degree in electronics from the Radio Engineering Department, HuaZhong (Central China) University of Science and Technology, Wuhan, China, in 1982 and the Ph.D. degree (with honors) from the Institut National Polytechnique of Toulouse, France, in 1988.

Since then he has been doing postdoctoral research in the Electronics Laboratory of ENSEEIHT in Toulouse, France. His main research interest is the application of various numerical methods to two- and three-dimensional boundary condition problems in electromagnetics, including propagation in inhomogeneous and anisotropical waveguides and planar waveguides, discontinuity modeling in scattering and radiating structures, and the design of microwave and millimeter-wave devices.

Dr. Tao received a Chinese Overseas Graduate Scholarship in 1982. His Ph.D. dissertation received the Leopold Escande Prize in 1989.



Henri Baudrand (M'86–SM'90) was born in France in 1939. He received the Diplome d'Ingénieur degree in electronics and the Docteur-es-Science degree in microwaves, both from the Institut National Polytechnique of Toulouse, France, in 1962 and 1966 respectively.

Since then he has been working on the modeling of active and passive microwave circuit devices in the Electronics Laboratory of ENSEEIHT in Toulouse. Currently he is a Professor of Microwaves and is in charge of the Microwave Research Group.